Performance Evaluation of an Adaptive PD Robot Controller

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Abstract

This paper presents a study of an adaptive controller presented by Tomei in [1]. The proposed compensator is a PD controller algorithm for point-to-point control that is adaptive with respect to the gravity parameters of robot manipulators.

Initially, we review the use of PD controllers for point-to-point robot movement. First, we discuss PD controllers; both with and without gravity compensation. This is followed by a discussion of PD controllers with imperfect gravity compensation. The deficiencies of the above controllers serve to motivate the discussion of Tomei's PD controller with adaptive gravity compensation for point-to-point movements.

Simulation studies of a $2R$ planar robot are presented. General conclusions on the utility of using Tomei's adaptive PD controller for point-to-point robot movements are arrived at through the simulation studies.

Introduction

The general equations of motion for a robot can be written as:

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \qquad (1)$$

e:

:) = $(n \times n)$ positive definite symmetric mass matrix

ı) = $(n \times n)$ coriolis and centripetal coefficients

ɪ) = $(n \times 1)$ vector of gravity terms

τ = $(n \times 1)$ vector of joint torques

q = $(n \times 1)$ vector of joint variables

n = number of joints

The general robot control problem considered here may be stated as: find the required joint torques, $\mathbf{u} = \boldsymbol{\tau}$, to achieve some desired motion: $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})_{desired}$.

Numerous control strategies have been proposed to realize desired motions of robots. Among these are: computed torque control [2], PD+ [3], PID [4], and PD with adaptive feedforward [5]. The purpose of this study is to analyze the performance of the adaptive PD controller proposed in [1].

2 Robot System Model

Throughout this work we will discuss the performance of different control algorithms. Numerical simulations of a planar $2R$ robot, see Fig. 1, using the varied controllers will be presented. The purpose of this study is to analyze the performance of the controllers proposed by in [1]. In his paper Tomei presents simulation results of a $3R$ robot with link lengths of 0.5 (m) and a payload of 5 (kg). Large controller gains were used in the simulation; $K_p = diag[10000]$ and $K_d = diag[3000]$ and the set points were not given. With these large gains, such a small payload when compared to robot, and the unknown set points(the effect of gravity on the system is greatly dependent on the set points) it is likely that the inertia, coriolis and centripetal terms dominated the dynamics of the simulation. Hence, the simulation is not likely a good measure of the performance of the proposed PD controller which is adaptive with respect to the gravity parameters.

The simulation data used in this work are listed below. The equations of motion of the robot arm are reported in [2]. The robotic system's parameter values are: $l_1 = l_2 = 1.0(m)$, $lc_1 = lc_2 = 0.5(m)$, and $m_1 = m_2 = 50.0(kg)$. The start position is $(\theta_1 = -\pi/2, \theta_2 = -\pi/2)$ and the goal position is $(\theta_1 = 0, \theta_2 = \pi/2)$. In order to facilitate controller comparison the same gains and start and goal positions

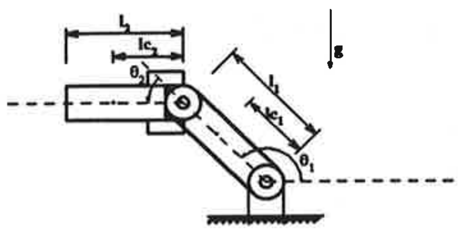

Figure 1: $2R$ Planar Robot

will be used throughout the paper. Moreover, realizable gains have been chosen to simulate a more practical robot model. The gains used were; $K_p = 200I$ and $K_d = 1000I$, where I is a (2×2) identity matrix. For the adaptive controllers, the initial parameter estimates used were 1/2 their actual value.

3 PD Set-Point Control

PD, or proportional plus derivative, control may be used for the set-point, or point-to-point, control of robots. Set-point control is used to move the robot from one prescribed configuration to another regardless of the required motion between the set-points.

3.1 Simple PD Control

An independent joint PD control to achieve the set-point $\mathbf{q}_{desired}$ is:

$$\mathbf{u} = -K_p\tilde{\mathbf{q}} - K_d\dot{\tilde{\mathbf{q}}} \qquad (2)$$

where:

$$\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_{desired}$$
$$\dot{\tilde{\mathbf{q}}} = \dot{\mathbf{q}}$$

It can be shown that this control law, Eq. 2, in the absence of gravity achieves zero steady-state error, see [2]. We now study the effectiveness of Eq. 2 in the presence of gravity.

In the presence of gravity the controller given by Eq. 2 will achieve some steady-state robot configuration $\mathbf{q}^*$. From Eq. 1 and Eq. 2 we see that $\mathbf{q}^*$ must satisfy:

$$-K_p(\mathbf{q}^* - \mathbf{q}_{desired}) = \mathbf{g}(\mathbf{q}^*) \qquad (3)$$

where: $(\mathbf{q}^* - \mathbf{q}_{desired})$ is the steady-state error. Eq. 3 can be physically interpreted as yielding the equilibrium configuration, $\mathbf{q}^*$, that must be reached such that

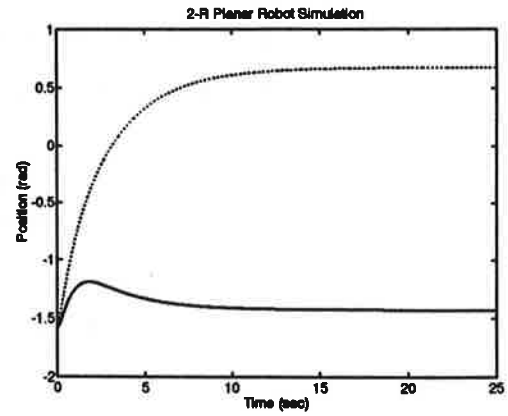

Figure 2: Simple PD Control

the joint motors generate a steady-state holding torque sufficient to balance the gravity torque, $\mathbf{g}(\mathbf{q}^*)$. We see, from Eq. 3, that to reduce the steady-state error the position gain K_p should be increased.

Another interpretation of the control law given by Eq. 2 follows. We assume the robot has achieved the desired steady-state configuration, $\tilde{\mathbf{q}} = 0 (\mathbf{q} = \mathbf{q}_{desired})$, and $\dot{\mathbf{q}} = 0$. From the control law, Eq. 2, we yield the control vector $\mathbf{u} = 0$. In the presence of gravity, with this control, the robot arm will immediately begin to fall. Therefore, the desired configuration $\mathbf{q} = \mathbf{q}_{desired}$ can never be attained.

3.1.1 Simulation: Simple PD Control

A simulation of the robotic system using the PD control given by Eq. 2 in the presence of gravity was performed. The joint angles can be seen in Fig. 2. Note the large steady-state error; each joint angle error ceeds 50(deg). As the above analysis predicted, d the lack of proper gravity compensation, and the relatively small gains used, large steady-state errors have occurred.

3.1.2 PD Control with Gravity Compensa

In order to avoid steady state-errors the indepen joint PD control can be augmented with gravity pensation:

$$\mathbf{u} = -K_p\tilde{\mathbf{q}} - K_d\dot{\tilde{\mathbf{q}}} + \mathbf{g}(\mathbf{q}) \qquad (4)$$

The gravity compensation negates the effect of gravity. The remaining PD terms in the control law, Eq. 4, are then essentially controlling a "weightless" robot. The modified control law, Eq. 4, realizes the same control as Eq. 2, therefore, it too achieves zero steady-state error.

4 PD Control with Adaptive Gravity Compensation

In using PD control with gravity compensation a problem arises because $\mathbf{g}(\mathbf{q})$ is a physical quantity which can only be approximated and we have demonstrated that any error in $\mathbf{g}(\mathbf{q})$ will cause a steady-state position error. In order to avoid any steady-state error in set-point tracking with PD control Tomei proposes a PD controller that is adaptive with respect to the gravity term, $\mathbf{g}(\mathbf{q})$. Commonly PID controllers, which require the addition of n integrators to the system, are used to eliminate steady-state position errors, see [6] and [2]. However, using the adaptive PD controller when the mass of the payload is unknown requires the introduction of only one integrator.

We proceed by factoring the gravity terms into known and estimated values. We let $\mathbf{g}(\mathbf{q}) = G(\mathbf{q})\mathbf{p}$ where: $G(\mathbf{q}) = (n \times m)$ known parameter matrix and $\mathbf{p} = (m \times 1)$ unknown constant parameter vector. The adaptive controller uses $\hat{\mathbf{p}}$, an estimate of $\mathbf{p}$, to compute the gravity compensation term. The control law is given by:

$$\mathbf{u} = -K_p\tilde{\mathbf{q}} - K_d\dot{\tilde{\mathbf{q}}} + G(\mathbf{q})\hat{\mathbf{p}} \tag{5}$$

where:

$$\dot{\hat{\mathbf{p}}} = -\beta G^T(\mathbf{q})\left[\gamma\dot{\mathbf{q}} + \frac{2\tilde{\mathbf{q}}}{1+2\tilde{\mathbf{q}}^T\tilde{\mathbf{q}}}\right] \tag{6}$$

β is a positive constant and γ is a positive constant such that:

$$\begin{aligned} \gamma &> \frac{2\lambda_M(M)}{\sqrt{\lambda_m(M)\lambda_m(K_p)}} \\ \gamma &> \frac{1}{\lambda_m(K_d)}\left[\frac{\lambda_M(K_d)^2}{2\lambda_m(K_p)} + 4\lambda_M(M) + \frac{k_c}{\sqrt{2}}\right] \end{aligned} \tag{7}$$

where, $\lambda_M(M)$ and $\lambda_m(M)$ are the maximum and minimum eigenvalues of M and k_c is a positive constant such that $||C(\mathbf{q}, \dot{\mathbf{q}})|| \leq k_c||\dot{\mathbf{q}}||$. Notice that the rate of adaptation is controlled by the choice of β. A choice of large β enables the controller to adapt faster. However, this will produce large changes in the estimate $\hat{\mathbf{p}}$ and may lead to "choppy" control torques.

By adding the adaptive parameter, $\hat{\mathbf{p}}$, we have increased the order of the system by m. In the case that only the payload of the system is unknown one integrator is sufficient to implement the adaptive controller. For a proof that the controller, Eq. 5, is globally asymptotically stable see [1].

Variable	Steady-State Error
q_1	0.0231 (deg)
q_2	0.3674 (deg)
p	8.4329×10^{-4} (kg)

Table 1: Adaptive PD Control Steady-State Errors

4.1 Simulation: Adaptive PD Control

For our $2R$ robot the mass m_2 was assumed unknown. Therefore,

$$\mathbf{p} = [m_2] \tag{8}$$

Note that with only one unknown parameter the vector $\mathbf{p}$ becomes a scalar. We rewrite the gravity term, $G(\mathbf{q})\hat{\mathbf{p}}$ as:

$$G(\mathbf{q})\hat{\mathbf{p}} = \mathbf{g}_{known}(\mathbf{q}) + \mathbf{g}_{adaptive}(\mathbf{q}) \tag{9}$$

where:

$$\mathbf{g}_{known}(\mathbf{q}) = g\begin{bmatrix} m_1 l_{c1}\cos(\theta_1) \\ 0 \end{bmatrix} \tag{10}$$

$$\mathbf{g}_{adaptive}(\mathbf{q}) = G(\mathbf{q})\hat{\mathbf{p}} \tag{11}$$

and:

$$G(\mathbf{q}) = g\begin{bmatrix} l_1\cos(\theta_1) + l_{c2}\cos(\theta_1+\theta_2) \\ l_{c2}\cos(\theta_2) \end{bmatrix} \tag{12}$$

For the trajectory studied $\min(k_c) = 41$ which resulted in $\min(\gamma) = 8.6$. The constants used were $\beta = 100.0$ and $\gamma = 8.6$. The joint angles are plotted in Fig. 3; θ_1 solid, θ_2 dotted. The required control torques are plotted in Fig. 4. The adaptive parameter, m_2, is shown in Fig. 5. The steady-state errors are listed in Tbl. 1. The desired set-point was achieved with minimal steady-state error. Furthermore, note how well the unknown mass of the payload was "learned" and the large settling time of the response.

An additional simulation was performed to demonstrate how quickly the adaptive PD control learns the unknown parameter when the desired motion is to just maintain a set-point. The set-point studied was $\theta_1 = \theta_2 = 0$. The adaptive parameter, m_2 is shown in Fig. 6. Notice how fast the unknown mass m_2 was learned.

5 Conclusions

Tomei's adaptive PD set-point controller was studied and its utility was verified through simulation of a $2R$

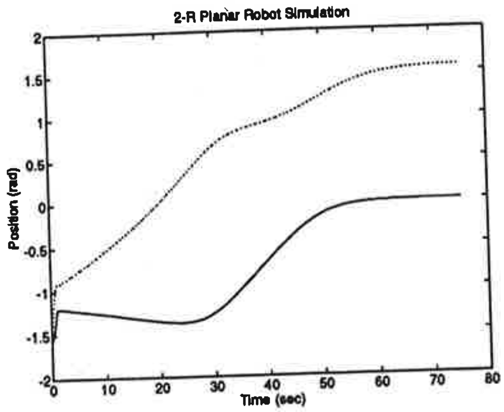

Figure 3: Adaptive PD Control: Joint Angles

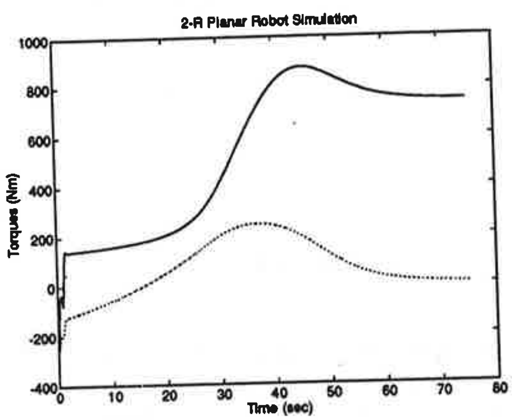

Figure 4: Adaptive PD Control: Control Torques

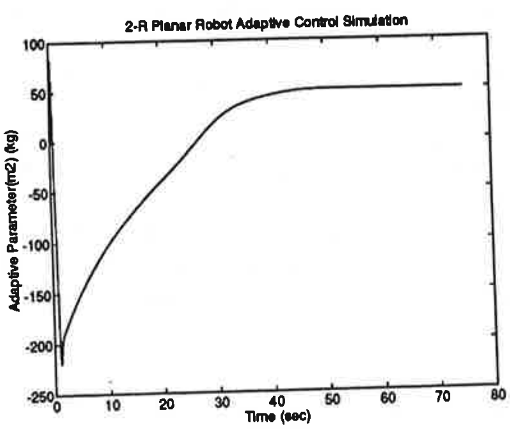

Figure 5: Adaptive PD Control: Param. Est. $\hat{\mathbf{p}}$

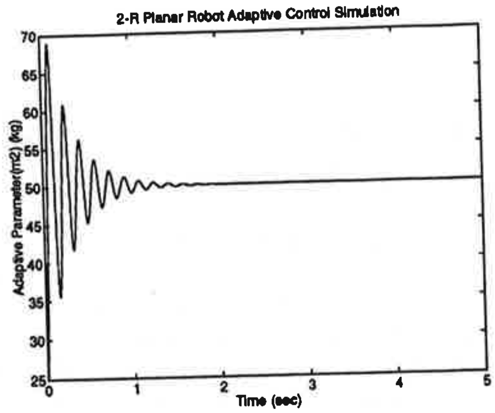

Figure 6: Adaptive PD Control: Param. Est. $\hat{\mathbf{p}}$

planar robotic system. We noted a slow response of the adaptive PD controller for set point control. However, when used to maintain a set point the controller performs well and learns the unknown parameter very quickly.

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